

Finite Tests from Functional Characterizations*

Charles Gauthier[†] Raghav Malhotra[‡] Agustín Troccoli Moretti[§]

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Abstract

Classically, testing whether decision makers belong to specific preference classes involves two main approaches. The first, known as the functional approach, assumes access to an entire demand function. The second, the revealed preference approach, constructs inequalities to test finite demand data. This paper bridges these methods by using the functional approach to test finite data through preference learnability results. We develop a computationally efficient algorithm that generates tests for choice data based on functional characterizations of preference families. We provide these restrictions for various applications, including homothetic and weakly separable preferences, where the latter’s revealed preference characterization is provably NP-Hard. We also address choice under uncertainty, offering tests for betweenness preferences. Lastly, we perform a simulation exercise demonstrating that our tests are effective in finite samples and accurately reject demands not belonging to a specified class.

Keywords: revealed preference, finite data, preferences, weak separability

JEL classification: C12, C91, D0, D11, D12

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[†]KU Leuven; charles.gauthier@kuleuven.be

[‡]University of Leicester; r.malhotra@leicester.ac.uk

[§]University of Warwick; agustin.troccoli-moretti@warwick.ac.uk

1 Introduction

In empirical analysis of decision making, analysts frequently encounter the challenge of verifying whether a rational decision maker (DM) adheres to a specific class of choice behavior from finite choice data.¹ Such tests are generally conducted using the revealed preference (RP) approach that involves checking the feasibility of a system of inequalities. However, for some preference classes revealed preference tests may not be tractable. In contrast, testing if the DM’s entire demand function belongs to these classes may be relatively simple.² This paper provides an alternative to the revealed preference paradigm by adapting tests of demand functions to finite choice data, allowing us to exploit the gap in simplicity between RP and functional tests.

We show how to use recent results on preference learnability ([Beigman and Vohra, 2006](#)) to adapt functional tests to finite datasets.³ We depart from the RP tradition by shifting the focus from testing the dataset itself to testing the DM. An RP test always rejects a dataset if no preference from the class of interest can generate observed demands and never rejects a dataset if there is a preference from the class of interest that rationalizes the data. In contrast, our tests only guarantee that the probability of a false rejection or a false acceptance can be made arbitrarily small as a computable function of the sample size. In return for these potential errors, we are able to unite functional and finite data testing approaches and gain tractability.

Classical results on learnability show that if price-demand pairs are generated from a preference relation, then any rationalizing preference converges to the true underlying preference relation ([Mas-Colell, 1977](#)). A naive approach would thus sample data, pick any preference that generates a demand function rationalizing the data, and test whether the demand function is from some class of preferences. Suppose that we follow this procedure and find that a rationalizing demand function is from the class of interest. Since we did not check every rationalizing demand function, we cannot say that the true demand is from this class. Suppose, instead, we find a rationalizing demand function that is not from the class of interest. Again, since we did not check every rationalizing demand function, we cannot say that the true demand is not from

¹For example, verifying DMs for weak separability or expected utility.

²Weak separability is an example of one such class; the test of the demand function is easy to conduct, but the RP characterization is known to be NP-Hard ([Echenique, 2014](#)).

³[Chambers et al. \(2021\)](#) prove similar finite sample extrapolation results when the analyst observes binary comparisons.

this class.⁴

The previous discussion shows that no conclusion can be drawn about whether the true preference relation belongs to some class of interest unless we test every rationalizing demand. The results on learnability allow us to circumvent this issue by computing an ex-post bound⁵ on how far *any* rationalizing demand can be from the true demand.⁶ We then show that if the rationalizing preference is sufficiently far from a given class of preferences, then we can infer that the true preference is not from the said class with arbitrary certainty. We heavily borrow from the Probably Approximately Correct (PAC) learning framework and thus call our tests probably approximately correct tests. These are in contrast to revealed preference tests that are exact and do not involve any probabilistic statement.⁷⁸

We now give a short outline of our main argument. Let \mathcal{M} denote the set of all “rational” preferences. Suppose a decision maker has a preference $\succeq \in \mathcal{M}$ with corresponding demand function $x_{\succeq}(p, I)$. In what follows, we may omit the dependence of demands on prices p and income I . We wish to test if \succeq belongs to some class of preferences $\mathcal{C} \subset \mathcal{M}$. We assume the analyst has access to a uniformly continuous *functional restriction* $\mathcal{R} : \mathcal{M} \rightarrow \mathbb{R}$ such that $\mathcal{R}(x_{\succeq}) = 0$ if and only if $\succeq \in \mathcal{C}$.

By uniform continuity, there exists a smooth monotone function $\gamma : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ with $\gamma(0) = 0$ such that for all $\succeq, \succeq' \in \mathcal{M}$

$$|\mathcal{R}(x_{\succeq}) - \mathcal{R}(x_{\succeq'})| \leq \gamma \left(\int_{\mathcal{P}} |x_{\succeq}(p) - x_{\succeq'}(p)| dp \right).$$

Suppose the data are generated by the true demand function x_{\succeq^*} . After sampling some dataset, the analyst constructs a rationalizing demand function x_{\succeq} . Assume for now that we can infer that the generalization error $erf(\succeq, \succeq^*) := \int_{\mathcal{P}} |x_{\succeq}(p) - x_{\succeq^*}(p)| dp$ of

⁴This problem relates to the notion of recursive enumerability (Chambers, Echenique, and Shmaya, 2017) in that (in)consistency with the theory cannot be shown in a finite number of steps.

⁵By ex-post bound, we mean that the bound depends on the dataset the analyst samples.

⁶Beigman and Vohra (2006) show that such bound can be computed efficiently under reasonable conditions on preferences.

⁷Revealed preference tests return a binary outcome $\{0, 1\}$. The dataset is either rejected 0 or not rejected 1.

⁸To our knowledge, the only other paper that constructs a test of finite data from a procedure relating to derivatives is Aguiar and Serrano (2018). They use finite data to extend the measure of irrationality for demand functions defined in Aguiar and Serrano (2017).

\succeq is at most ϵ .⁹ From the above, we have

$$\mathcal{R}(x_{\succeq}) - \mathcal{R}(x_{\succeq^*}) \leq \gamma(\text{erf}(\succeq, \succeq^*)) \implies \mathcal{R}(x_{\succeq^*}) \geq \mathcal{R}(x_{\succeq}) - \gamma(\epsilon).$$

Observe that $\mathcal{R}(x_{\succeq^*}) > 0$ whenever $\mathcal{R}(x_{\succeq}) > \gamma(\epsilon)$. Thus, if the analyst observes $\mathcal{R}(x_{\succeq}) > \gamma(\epsilon)$, he can infer $\mathcal{R}(x_{\succeq^*}) \neq 0$ such that $\succeq^* \notin \mathcal{C}$.

Next, by [Mas-Colell \(1977\)](#) the rationalizing demand function $x_n(p, I)$ converges to the true demand x_{\succeq^*} as n grows, where n denotes the size of the dataset. Therefore, by continuity, we have $\{\mathcal{R}(x_{\succeq_n})\}_{n=1}^{\infty} \rightarrow \mathcal{R}(x_{\succeq^*})$. We can show that under suitable assumptions, the ex-post bound $\{\epsilon_n\}_{n=1}^{\infty}$ converges to zero with probability greater than $1 - \delta$ for any $\delta > 0$. Thus, by continuity, we have $\{\gamma(\epsilon_n)\}_{n=1}^{\infty} \rightarrow 0$. In other words, if the demands do not fall in the class of preferences \mathcal{C} , for every $\delta > 0$, there is $n_0 \in \mathbb{N}$ such that $\mathcal{R}(x_{\succeq_n}) > \gamma(\epsilon_n)$ for all $n \geq n_0$ with probability greater than $1 - \delta$. Hence, we can conclude that $\mathcal{R}(x_{\succeq^*}) > 0$ such that $\succeq^* \notin \mathcal{C}$ with probability $1 - \delta$.

In summary, if we can infer that the maximum error is less than ϵ with probability $1 - \delta$, our tests possess the following properties:

1. A dataset that can be rationalized by a utility function from a class of interest is rejected with probability less than δ .
2. A dataset that cannot be rationalized by a utility function from a class of interest is rejected with probability $1 - \delta$.

These properties imply that our tests can never prove that a data set is consistent or inconsistent with the theory. Contrary to RP tests, we reject consistent datasets with probability approaching zero, and reject inconsistent datasets with probability approaching one. It is this difference in size and power that allows us to construct a polynomial time test for weak separability. In other words, the NP-Hardness of testing weak separability ([Echenique, 2014](#)) is a feature of the knife-edge nature of RP tests.

Recent results on preference learnability ([Beigman and Vohra, 2006](#)) show that we can infer that the maximum generalization error is less than ϵ with probability $1 - \delta$, where delta is a function of the sample. Our procedure depends purely on PAC learnability and abstracts from the specific nature of the optimization problem of the agent and the family of choice sets observed by the analyst. As such, it is applicable to choice under risk and pairwise comparisons, among others. Perhaps

⁹This is an “ex-post error bound”.

counterintuitively, our procedure can easily accommodate measurement error as shown by [Bartlett et al. \(1994\)](#). All we require is for the class of rational demands to have a finite fat-shattering dimension, a condition that is satisfied for the class of income-Lipschitz demand functions.¹⁰

To operationalize our approach, we construct functional restrictions for a variety of classes of demands. First, we consider demands generated by the class of homothetic and weakly separable preferences. Then, we show that our procedure can be used to test whether demands are substitutes or complements. Lastly, we consider choice under uncertainty and develop a test that accepts all preferences from the betweenness class ([Dekel, 1986](#); [Chew, 1989](#)) but is stronger than any test known in the literature.

We implement our algorithm for homotheticity and weak separability via a simulation exercise. For practical purposes, we use the common AIDS demand model as it provides a first order approximation to the set of all rational demand functions. Note that this parametric specification is not required for our theoretical results, however. Our simulations show that our tests perform well in finite samples and correctly reject demands that are not homothetic and weakly separable. Therefore, our approach may have several applications where analysts often rely on heuristic methods to overcome computational complexities, such as in [Varian \(1983\)](#) and [Fleissig and Whitney \(2003\)](#).

Section 2 describes our setup and notation. Section 3 highlights how to construct our tests if the analyst has access to a functional characterization for a class of preferences. Section 4 obtains functional characterizations for common classes of preferences, substitutes and complements, and choice under uncertainty. Section 5 evaluates the empirical performance of our approach for the classes of homothetic and weakly separable preferences. Section 6 studies extensions of our approach, such as the presence of measurement error. Section 7 concludes.

1.1 Related Literature

The functional approach to testing decision-makers originated in the work of [Slutsky \(1915\)](#) and [Antonelli \(1971\)](#) who derived necessary and sufficient conditions for demand functions to arise from utility maximization.¹¹ For a detailed review of functional tests for classes of preferences, see [Deaton and Muellbauer \(1980\)](#). Since these tests rely on

¹⁰The fat-shattering dimension is to real-valued functions what the VC dimension is to discrete-valued functions. In our setup, it provides a measure of complexity of the class of demand functions.

¹¹This is the so-called “integrability problem” for demand functions. For a review, see [Hurwicz and Uzawa \(1971\)](#)

the whole demand function, they require access to infinite data. We depart from this literature by extending those functional tests to finite data.

The closest work linking restrictions on demand functions and finite data is [Aguiar and Serrano \(2018\)](#). In their paper, they extend the measure of bounded rationality defined in [Aguiar and Serrano \(2017\)](#) based on the Slutsky matrix to finite data. Their focus is on measuring and classifying bounded rationality with finite price-demand data and linear budgets. In contrast, our approach provides tests for general restrictions under preference monotonicity that is applicable to broader datasets such as pairwise comparisons.

The RP approach is standard to test the consistency of a finite dataset with the utility maximization hypothesis. The approach originated in the work of [Samuelson \(1938, 1939, 1947, 1948\)](#) and was further refined by [Afriat \(1967\)](#) and [Varian \(1982\)](#). In particular, [Varian \(1982\)](#) established that the Generalized Axiom of Revealed Preference (GARP) is a necessary and sufficient condition for any finite dataset to be consistent with utility maximization. The appeal of GARP stems from the ease with which the rationality of a data set can be verified through simple algebraic inequalities.

Since those seminal contributions, RP theory has been exploring various extensions such as functional form restrictions and inter-temporal models. Significant contributions include [Kubler, Selden, and Wei \(2014\)](#) for expected utility with objective (known) probabilities and [Echenique and Saito \(2015\)](#) for subjective probabilities. A similar problem for “translation-invariant” preferences is considered in [Chambers, Echenique, and Saito \(2016\)](#). [Polisson, Quah, and Renou \(2020\)](#) give a general method to construct RP inequalities that applies to several classes of preferences over risk and uncertainty.¹²

The RP literature concentrates on exhaustive restrictions on finite datasets that characterize¹³ a model and largely uses the sufficiency of first-order conditions.¹⁴ In some instances, those tests can pose a significant computational challenge. Indeed, [Echenique \(2014\)](#) and [Cherchye, Demuynck, De Rock, and Hjertstrand \(2015\)](#) show that testing for weak separability is NP-Hard. Accordingly, these tests have to rely on non-polynomial time algorithms such as mixed integer programming ([Cherchye, Demuynck, De Rock, and Hjertstrand, 2015](#); [Hjertstrand, Swofford, and Whitney,](#)

¹²For recent extensive reviews, see [Crawford and De Rock \(2014\)](#); [Echenique \(2019\)](#); [Demuynck and Hjertstrand \(2019\)](#).

¹³See [Chambers and Echenique \(2016, p. xv\)](#).

¹⁴The only revealed preference test that is not a characterization which we are familiar with is for probabilistic sophistication ([Epstein, 2000](#)).

2020), an efficient approach for small to medium sized datasets.

The deterministic RP approach has been generalized to a stochastic environment with measurement error by [Aguiar and Kashaev \(2021\)](#). Despite its appeal, their framework inherits the computational burden of testing classes of preferences whose RP restrictions are NP-Hard. Indeed, their approach involves simulations that require solving a (possibly NP-Hard) problem many times rather than just once. Consequently, tractability issues are exacerbated in their stochastic setting.

2 Setup

We consider individual decision-making where the analyst only has access to finite choices made by the DM. The DM chooses l -dimensional bundles $x \in \mathcal{X} \subset \mathbb{R}_+^l$ according to some preference $\succeq \subseteq \mathcal{X} \times \mathcal{X}$, where \mathcal{X} is a compact convex set. We associate every preference with its graph and write $x \succeq y$ if $(x, y) \in \succeq$, to be read as x is “at least as good as” y . We refer to the set of complete, transitive, twice differentiable, strongly convex, and monotone preferences as $\mathcal{M} \subset 2^{\mathcal{X} \times \mathcal{X}}$.¹⁵

Let $\mathcal{P} \subset \mathbb{R}_{++}^l$ represent the set of possible prices. An individual faces a sequence of prices that are drawn from some fixed distribution. For a price $p \in \mathcal{P}$, and by normalizing income, the choice set is defined as

$$\mathbf{B}_p := \{y \in \mathcal{P} : p \cdot y \leq 1\}$$

A demand function $x : \mathcal{P} \rightarrow \mathbf{X}$ associates to each budget $\mathbf{B}_p \subset \mathbf{X}$ an element of that budget $x(p) \in \mathbf{B}_p$. A *dataset* is a collection of prices and demands denoted by $\{(p, x)_k\}_{k=1}^n$.

We say that a preference relation \succeq *rationalizes* the data $\{(p, x)_k\}_{k=1}^n$ if for all prices, the demand $x(p)$ dominates every other element $y \in \mathbf{B}_p$:

$$x(p) \succeq y \quad \forall y \in \mathbf{B}_p$$

Let $\mathcal{C} \subset \mathcal{M}$ be a class of preferences. We say that a dataset $\{(p, x)_k\}_{k=1}^n$ is \mathcal{C} -rationalizable if there exists $\succeq \in \mathcal{C}$ such that \succeq rationalizes $\{(p, x)_k\}_{k=1}^n$.

¹⁵Preferences are *strongly convex* just in case for any $x \neq y$, $x \succeq y \Rightarrow \alpha x + (1 - \alpha)y \succ y$ for all $\alpha \in (0, 1)$; while preferences are said to be *monotone* just in case for any $x \neq y$ with $x_i \geq y_i$ for all $i = 1, \dots, l$, then $x \succ y$.

We say that a preference $\succeq \in \mathcal{M}$ generates a demand function x_{\succeq} if for all prices, the demand $x_{\succeq}(p)$ dominates every other element $y \in \mathbf{B}_p$:

$$x_{\succeq}(p) \succeq y \quad \forall y \in \mathbf{B}_p$$

We refer to the data point generated by \succeq under choice set \mathbf{B}_p as $(p, x_{\succeq}(p))$. A dataset whose demands are generated by \succeq is a collection of data points $D = \{(p, x_{\succeq})_k\}_{k=1}^n$. A dataset D is \mathcal{C} -rationalizable if there exists $\succeq \in \mathcal{C}$ such that \succeq rationalizes D .

Probably Approximately Correct Tests

Let \mathcal{D} denote the set of all possible datasets. A test $\mathcal{T} : \mathcal{D} \rightarrow \{0, 1\}$ is a function that inputs observations and outputs 0 or 1. We say that a dataset $\{(p, x)_k\}_{k=1}^n$ passes a test \mathcal{T} if

$$\mathcal{T}(\{(p, x)_k\}_{k=1}^n) = 1,$$

and fails a test \mathcal{T} if

$$\mathcal{T}(\{(p, x)_k\}_{k=1}^n) = 0.¹⁶$$

We now define desirable properties of a test.

Definition 1 (δ -Soundness). Let $\delta \in (0, 1)$. A test \mathcal{T} is sound with respect to the class \mathcal{C} if:

$$\mathcal{T}(\{(p, x)_k\}_{k=1}^n) = 0 \text{ implies } \mathbb{P}[\{(p, x)_k\}_{k=1}^n \text{ is } \mathcal{C}\text{-rationalizable}] \leq \delta.$$

In words, δ -soundness means that if the data fail the test, then the probability of a false rejection is smaller than δ . Hence, a test is sound if its size is bounded by some exogenously given δ .

Let \mathcal{B}_p denote a family of prices and σ denote a probability measure over this family. Suppose $\succeq \in \mathcal{M}$ and $\{(p, x)_k\}_{k=1}^n$ is the dataset obtained from i.i.d. draws of prices from measure σ . We denote the product measure over a dataset of size n by σ^n .

Definition 2 (Asymptotic Completeness). Let $\mathcal{C} \subset \mathcal{M}$ be a class of preferences. We call a test \mathcal{T} asymptotically complete with respect to the class \mathcal{C} and measure σ if for

¹⁶By accepting, we mean what is conventionally termed as failing to reject.

any preference $\succeq \notin \mathcal{C}$:

$$\lim_{n \rightarrow \infty} \sigma^n [\{(\mathbf{B}_p)_k\}_{k=1}^n \text{ such that } \mathcal{T}(\{(p, x_{\succeq})_k\}_{k=1}^n) = 0] \rightarrow 1.$$

In words, asymptotic completeness means that the test correctly rejects data that are not \mathcal{C} -rationalizable with probability approaching one as the size of the dataset increases. Hence, a test is asymptotically complete if its power goes to one.

Definition 3 (PAC Test). A test \mathcal{T} is Probably Approximately Correct (PAC) for the class \mathcal{C} given a distribution σ over prices and $\delta \in (0, 1)$ if there is a sample size $n(\delta)$ such that for any dataset $\{(p, x_{\succeq})_k\}_{k=1}^n$ with $n \geq n(\delta)$, the test \mathcal{T} is δ -sound and asymptotically complete.

2.1 Preference Learnability

We now state our main result that we use to construct our procedure. Let σ be a measure over the set of prices \mathcal{P} . Define the generalization error between two demand functions $x_{\succeq}, x_{\succeq'} : \mathcal{P} \rightarrow \mathcal{X}$ that arises from preferences \succeq, \succeq' as $erf_{\sigma}(\succeq, \succeq') = \int_{\mathcal{P}} |x_{\succeq}(p) - x_{\succeq'}(p)|^2 d\sigma$.

Theorem 2.1 ((Beigman and Vohra, 2006)). *Let \mathcal{C}_L be the set of all income Lipschitz demands with constant L . For any $x_{\succeq} \in \mathcal{C}_L$, $\epsilon, \delta > 0$ and measure σ , there exists an algorithm that for a number of observations $n_L(\epsilon, \delta)$ polynomial in $\frac{1}{\epsilon}$ and $\frac{1}{\delta}$, outputs a demand function $x_{\succeq'}$ such that $erf_{\sigma}(\succeq, \succeq') \leq \epsilon$ with probability $1 - \delta$.*

This theorem allows us to learn the true demand function for any distribution σ . Beigman and Vohra (2006) also provide an efficient constructive algorithm to find the demand x' . If we further fix a uniform distribution over prices, Kubler, Malhotra, and Polemarchakis (2020) show that the assumption of a known Lipschitz constant can be relaxed, but they do not give an explicit procedure to compute the required constants.

3 Framework and Testing Procedure

This section highlights the difference between our setup and the classical revealed preference setup.

Revealed Preference Framework

We think of a test as a three-step procedure. First, the analyst is confronted by a dataset of prices and demands. Then, Nature picks a preference that rationalizes the data. Finally, the analyst tests whether Nature could have picked a preference $\succeq \in \mathcal{C}$ that rationalizes the data. The timing of the classical approach is depicted in Figure 1.

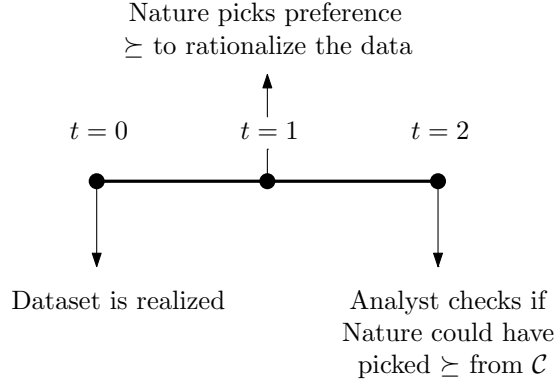


Figure 1: Depiction of the timing in the classic RP framework.

Preference Learnability Framework

In our setup, Nature first picks a preference. Then, the analyst samples prices from some fixed probability measure. This allows us to index our tests by the size of the dataset sampled. Finally, the analyst tests whether Nature picked $\succeq \in \mathcal{C}$ given the realized dataset. This allows us to make probabilistic statements about the belonging of the true preference to some class of preferences based on sampled data. Asymptotic completeness is informally defined as follows: *conditional* on Nature drawing a preference $\succeq \notin \mathcal{C}$, the **measure of datasets** rejected by our test goes to one as n goes to infinity. Thus, our approach tests Nature’s preferences rather than the dataset itself. The timing of our approach is depicted in Figure 2.

This shows the difference between the RP and the learnability approaches. Namely, our approach views the preference that Nature picks as fixed and learns about it from the data, thus allowing us to study asymptotics.

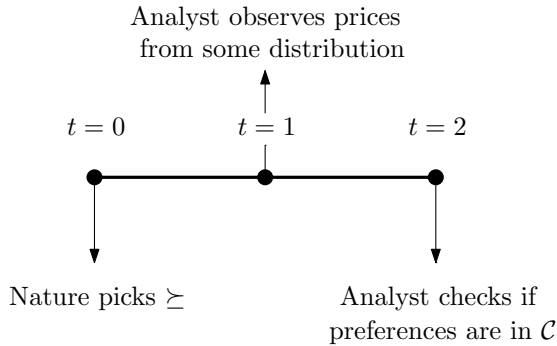


Figure 2: Depiction of the timing in our framework.

Functional Restriction and General Procedure

We call a function $\mathcal{R} : \mathcal{M} \times \mathcal{P} \rightarrow \mathbb{R}$ a *functional restriction* for a class of preferences $\mathcal{C} \subset \mathcal{M}$ if

$$\succeq \in \mathcal{C} \iff (\forall p \in \mathcal{P} \quad \mathcal{R}(\succeq)(p) = 0).$$

The functional restriction must hold at every point, so in a sense it is local and not global. The requirement for local restrictions inherently limits the scope of our testing capabilities. Specifically, this constraint prevents us from distinguishing between models that only exhibit differences at kinks. An example is the inability to differentiate between models such as expected utility and rank-dependent expected utility.

Despite this limitation, our approach remains capable of testing a broad range of models. For instance, we can effectively analyze Chew-Dekel preferences as outlined in [Dekel \(1986\)](#) and [Chew \(1989\)](#). These preferences result in first-order conditions that are linear in probabilities, allowing us to formulate applicable restrictions. We also describe functional restrictions for other classes of preferences such as homotheticity and weak separability in [Section 4](#).

As we informally argued in the Introduction, we need to put some discipline on our restrictions to bound the size of deviations as a function of the dataset assuming that the DM is truly from the class \mathcal{C} . We now define this property, which we refer to as uniform continuity.

Definition 4 (Uniform Continuity). Let σ be any distribution that admits a density. We call a restriction \mathcal{R} of class \mathcal{C} *uniformly continuous* at 0 if for every $\succeq \in \mathcal{M}$, $\succeq^* \in \mathcal{C}$,

and $\epsilon > 0$, there exists $\gamma(\epsilon) > 0$ such that:

$$erf_{\sigma}(\succeq^*, \succeq) < \gamma(\epsilon) \implies \left\{ \int_{\mathcal{P}} [\mathcal{R}(\succeq)(p) - \mathcal{R}(\succeq^*)(p)]^2 \right\}^{\frac{1}{2}} = \left\{ \int_{\mathcal{P}} [\mathcal{R}(\succeq)(p)]^2 \right\}^{\frac{1}{2}} < \epsilon.$$

This definition means that something *close to* a preference that satisfies our restrictions *almost* satisfies our restriction. In what follows, we show that if a functional restriction is uniformly continuous at zero, we can use it to construct a PAC test.

Remark. The function $\gamma(\epsilon)$ only depends on the underlying domain \mathcal{P} and the restriction chosen \mathcal{R} . It is independent of the underlying preference and sampling procedure.

3.1 Algorithmic Procedure

Let \mathcal{R} be a uniformly continuous restriction for a class $\mathcal{C} \subset \mathcal{M}$. We now further restrict the set \mathcal{M} to one with an income Lipschitz constant less than a known L . We propose a procedure to construct a PAC test.

1. Pick any sequence $\{\delta_k\}_{k=1}^{\infty}$ and $\{\epsilon_k\}_{k=1}^{\infty}$ such that $\delta_k, \epsilon_k \rightarrow 0$.
2. Given $\delta_k, \epsilon_k > 0$, pick n such that for any dataset $\{(p, x_{\succeq}(p))\}_{k=1}^n$, any rationalizing preference \succeq , and any true preference \succeq^* ,

$$erf_{\sigma}(\succeq, \succeq^*) \leq \epsilon_k$$

with probability greater than $1 - \delta_k$.

3. Compute the test statistic:

$$T_n = \gamma \left(\left\{ \int_{\mathcal{P}} [\mathcal{R}(\succeq)(p)]^2 d\sigma \right\}^{\frac{1}{2}} \right).$$

4. If $T_n > \epsilon_k$, reject the hypothesis that the true underlying preference \succeq^* is in \mathcal{C} .
Else, let $k := k + 1$ and go back to the second step.

The following result shows that a test based on the previous procedure indeed defines a PAC test.

Theorem 3.1. *Let \mathcal{T} be a test where $\mathcal{T}(D) = 0$ if the above procedure rejects D , and 1 otherwise. Then, \mathcal{T} is a PAC test for class \mathcal{C} .*

Proof. 1. We want to show that the algorithm leads to a δ -sound test. For the sake of a contradiction, suppose $\mathcal{T}(D) = 0 \iff T_n > \text{erf}(\succeq, \succeq^*)$ but $\succeq^* \in \mathcal{C}$. By construction, $\text{erf}(\succeq, \succeq^*) \leq \epsilon$ with probability $1 - \delta$. Hence, with probability $1 - \delta$:

$$\text{erf}(\succeq, \succeq^*) < T_n = \gamma \left(\left\{ \int_{\mathcal{P}} [\mathcal{R}(\succeq)(p) d\sigma]^2 \right\}^{\frac{1}{2}} \right)$$

By uniform continuity, this implies that

$$\left\{ \int_{\mathcal{P}} [\mathcal{R}(\succeq)(p) d\sigma]^2 \right\}^{\frac{1}{2}} < \left\{ \int_{\mathcal{P}} [\mathcal{R}(\succeq^*)(p) d\sigma]^2 \right\}^{\frac{1}{2}},$$

which is a contradiction. Therefore, $\succeq^* \notin \mathcal{C}$ with probability $1 - \delta$

2. We want to show that the algorithm leads to an asymptotically complete test. Suppose $\succeq^* \notin \mathcal{C}$. By construction,

$$\lim_{k \rightarrow \infty} \mathbb{P}[\text{erf}(\succeq, \succeq^*) > \epsilon] \rightarrow 0 \quad \text{for all } \epsilon, \delta > 0$$

By continuity of γ , as the size of the dataset increases and picks rationalizations \succeq^n ,

$$\gamma \left(\left\{ \int_{\mathcal{P}} [\mathcal{R}(\succeq_n)(p) d\sigma]^2 \right\}^{\frac{1}{2}} \right) \rightarrow \gamma \left(\left\{ \int_{\mathcal{P}} [\mathcal{R}(\succeq^*)(p) d\sigma]^2 \right\}^{\frac{1}{2}} \right) = \gamma^* > 0.$$

Thus, there must eventually be a n where with high probability,

$$\epsilon < \gamma \left(\left\{ \int_{\mathcal{P}} [\mathcal{R}(\succeq_n)(p) d\sigma]^2 \right\}^{\frac{1}{2}} \right),$$

which proves that $\succeq^* \notin \mathcal{C}$. □

4 Functional Restrictions

We first consider the class of demand functions generated by homothetic preferences and then the class of demand functions generated by weakly separable preferences

considered in [Goldman and Uzawa \(1964a\)](#).¹⁷ Next, we consider the class of demand functions that are complements and substitutes. Finally, we consider the class of demand functions generated by preferences in the betweenness class for choice under risk.

In what follows, we use derivatives of demand functions to construct functional restrictions. Thus, we first show that derivatives and their compositions generate uniformly continuous restrictions.

Lemma. *Suppose the set of preferences \mathcal{M} generates demands that are twice continuously differentiable. The map*

$$\mathcal{R}(x) := \frac{\partial}{\partial p} x(p, I)$$

is uniformly continuous.

Proof. The derivative map is linear, and hence continuous. By assumption, the derivative is differentiable. As the space of bundles is compact, the map is defined from the set of all differentiable functions over a compact set to itself. By the Arzelà-Ascoli theorem, this set of functions is pre-compact, therefore any continuous function to itself must also be uniformly continuous. \square

4.1 Homotheticity

Lemma. *Let $x_{\succeq}(p, I)$ be a demand function, \succeq is homothetic if and only if $\frac{d^2 x}{dI^2} = 0$.*

Proof. By definition, if $x(p, I)$ is homothetic,

$$x(p, I) = f(p) \times I \implies \frac{dx}{dI} = f(p) \implies \frac{d^2 x}{dI^2} = 0$$

Now suppose $\frac{d^2 x}{dI^2} = 0$ this implies

$$\frac{dx}{dI} = f(p) \implies x(p, I) = x(p, 0) + f(p)I.$$

We know that demand with 0 income must be 0, hence:

$$x(p, I) = f(p)I$$

¹⁷A similar analysis could be used to construct tests of strong and Pearce separability.

□

Corollary 1. $\mathcal{R}(\succeq)(p) = \frac{d^2x}{dI^2}$ is a functional characterization of homotheticity.

Lemma. $\mathcal{R}(\succeq)(p) = \frac{d^2x}{dI^2}$ is uniformly continuous over the space of smooth demand functions.

Proof. Again, continuity of \mathcal{R} follows from the second derivative being a linear transformation of functions. As preferences are assumed to be smooth and demand is defined in a compact domain, the space of demand functions is compact [Hildenbrand \(1970\)](#). □

Remark. More generally, if we are willing to assume that demand functions have bounded derivatives for both prices and incomes, the compactness of the space of demand functions over a compact set follows quite simply from the Arzelà–Ascoli theorem.

4.2 Weak Separability

Definition 5 (Weak Separability). Suppose that the set of goods can be divided into two subgroups, g_1 and g_2 . We say that a preference \succeq is weakly separable if there exist subutility functions v_1, v_2 for g_1, g_2 , respectively, and a macro utility function u such that $u(v_1(x_i), v_2(x_j))$ represents \succeq , where $i \in g_1$ and $j \in g_2$.

Definition 6 (Slutsky Matrix). The Slutsky Matrix of a demand function $x(p, I)$ is defined as

$$S = \frac{dx}{dp} + x \left[\frac{dx}{dI} \right]^\top.$$

We refer to the i_j^{th} term of this matrix as S_{ij} . The next result characterizes weak separability from restrictions on the Slutsky matrix.

Theorem 4.1 ([Goldman and Uzawa \(1964b\)](#)). Suppose the set of goods can be divided into two subgroups, g_1 and g_2 . The preference underlying a demand function $x(p, I)$ is weakly separable if and only if:

$$\forall i \in g_1, j \in g_2, \quad S_{ij} = K^{1,2}(x) \frac{\partial x_i}{\partial I} \frac{\partial x_j}{\partial I}.$$

We now adapt this theorem to construct a test of weak separability in the case of 3 goods, where $g_1 = \{1\}$ and $g_2 = \{2, 3\}$. To this end, notice that with 3 goods the only Slutsky terms that need to be checked are S_{12} and S_{13} .

Lemma. *The demand function with 3 goods and subgroups $g_1 = \{1\}$, $g_2 = \{2, 3\}$ can be expressed by a weakly separable utility function if and only if for all prices and incomes:*

$$\frac{S_{12}}{S_{13}} - \frac{\frac{\partial x_2}{\partial I}}{\frac{\partial x_3}{\partial I}} = 0.$$

This result follows directly from the above theorem and the only possible pairs of goods being (x_1, x_2) and (x_1, x_3) . We can now define a functional characterization for weakly separable demand functions.

Corollary 2. *A functional characterization of weak separability is given by*

$$\mathcal{R}(\succeq)(p) = \frac{S_{12}}{S_{13}} - \frac{\frac{\partial x_2}{\partial I}}{\frac{\partial x_3}{\partial I}}.$$

Finally, note that this functional characterization is uniformly continuous as it is a function of derivatives and compositions of uniformly continuous functions.

4.3 Complementarity and Substitutability

Although intuitive notions of complementarity and substitutability might seem straightforward, there have been multiple attempts to formalize these concepts, each having advantages and limitations (see [Samuelson \(1974\)](#) or [Newman \(1987\)](#) for a survey).

Among the definitions looking at demand functions¹⁸, perhaps the most intuitive one is the notion of *gross complementarity (substitutability)*, requiring goods i and j to be gross complements (substitutes) whenever $\frac{\partial x_i(p, I)}{\partial p_j} < (>) 0$. Due to the existence of income effects, the gross notion of complementarity is not symmetric.

For this reason, the alternative notion of *net complementarity*, due to [Hicks and Allen \(1934\)](#), defines it in terms of compensated demands. That is, for an underlying utility function u , good i is a net complement (substitute) of good j if $\frac{\partial h_i(p, u)}{\partial p_j} < (>) 0$, where $h(p, u)$ is the Hicksian or compensated demand. By virtue of the fact that $S_{ij} := \frac{h_i}{p_j}$, and that the Slutsky matrix is symmetric, unlike its gross counterpart, the notion of net complementarity is symmetric. However, as pointed out by [Samuelson](#)

¹⁸The first attempts to define complements and substitutes, originally due to [Auspitz and Lieben \(1889\)](#) and later by [Fisher \(1892\)](#), [Edgeworth \(1897\)](#) and [Pareto \(1909\)](#), were based on the signs of the second derivatives of the utility function, whenever they exist. Given the well-known limitation of this approach that second derivatives are not invariant to monotone transformations, we focus on definitions using demand functions.

(1974), this notion lacks intuitive appeal. While a valid criticism, the Hicks-Allen notion has become the most widely used in economics.¹⁹

For any $t \in \mathbb{R}$, let

$$F^{>}(t) = \begin{cases} 0 & \text{if } t > 0 \\ 1 & \text{otherwise} \end{cases} \quad \text{and} \quad F^{<}(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{otherwise} \end{cases}.$$

Proposition 1. *Given a demand function $x(p, I)$, a functional restriction of gross complementarity (substitutability) is given by*

$$\mathcal{R}(\succeq)(p) = F^{>(<)} \left(\frac{\partial x_i(p, I)}{\partial p_j} \right),$$

while a functional restriction for net complementarity (substitutability) is given by

$$\mathcal{R}(\succeq)(p) = F^{>(<)} \left(\frac{\partial x_i(p, I)}{\partial p_j} + x_j(p, I) \cdot \frac{\partial x_i(p, I)}{\partial I} \right).$$

Note that, as in the case of weak separability, these functional restrictions are uniformly continuous except at zero as they are functions of derivatives of uniformly continuous functions. Hence, they can be used to test for strict complementarity and strict substitutability.

4.4 Choice under Risk

In this section, we study demand for contingent claims in the setting of Kubler et al. (2014) that gives a functional characterization for expected utility preferences.²⁰ We extend the analysis to utility functions belonging to the betweenness class as characterized by Dekel (1986) and Chew (1989). A preference in the betweenness class admits an implicit representation and generalizes the expected utility model by requiring indifference curves to be straight lines, but not necessarily parallel.

There are S states of nature, with elements $s \in \{1, 2, \dots, S\}$. The DM has preferences over state-contingent consumption $\mathbf{x} \in \mathbb{R}_{++}^S$ and objective beliefs $\pi = (\pi_1, \dots, \pi_S) \in [0, 1]^S$, where $\sum_{s \in S} \pi_s = 1$. We assume that the agent's preferences

¹⁹See Weinstein (2022) for a novel treatment of complementarity and substitutability aimed to address Samuelson's criticism.

²⁰See Theorem 3, pg 3475 Kubler et al. (2014).

can be represented by a strictly increasing, three times continuously differentiable, and strictly quasi-concave utility function $V(\mathbf{x}; \pi)$. Note that π enters as a parameter rather than a choice variable. By [Dekel \(1986\)](#) (p. 312), a preference over lotteries belongs to the betweenness class if and only if it admits a representation $V(\mathbf{x}; \pi)$ such that:

$$V(\mathbf{x}; \pi) = \sum_{s \in S} \pi_s u(x_s, V(\mathbf{x}; \pi)), \quad (1)$$

where $u : \mathbb{R}_{++} \times [0, 1] \rightarrow \mathbb{R}$ is a Bernoulli utility index that is increasing in the first argument and continuous in the second. We further assume that u is three times continuously differentiable in both arguments and concave in consumption. We denote the derivative of u with respect to the i -th argument by u_i and require $u_{11} < 0$.

Markets are assumed to be complete, with prices p_s for each $s \in \{1, 2, \dots, S\}$. As discussed in [Kubler et al. \(2014\)](#), this assumption allows us to focus directly on the contingent-claim demand $\mathbf{x}(\mathbf{p}, I, \pi)$ that solves

$$\max_{\mathbf{x} \in \mathbb{R}_{++}^S} V(\mathbf{x}, \pi) \quad \text{subject to} \quad \langle \mathbf{p}, \mathbf{x} \rangle \leq I. \quad (2)$$

Proposition 2. *Suppose $S > 2$ and contingent demands $x_s(\mathbf{p}, I, \pi) > 0$, $s = 1, \dots, S$, can be rationalised by a well-defined utility function. Then, a necessary condition for this utility function to be in the betweenness class defined in equation (1) is the existence of a function $f : \mathbb{R}_{++}^3 \rightarrow \mathbb{R}$ such that $x_s(\mathbf{p}, I, \pi) = f(x_1, x_2, k_s)$, where $k_s = \frac{\pi_s p_1}{\pi_1 p_s}$, f is strictly increasing in k_s , and $f(x, x, 1) = x$.*

Proof. The first-order conditions of the maximization problem (2) for any state $s \in \{2, 3, \dots, S\}$ yield the following results:

$$\text{MRS}_{1,s} = \frac{\partial V(\mathbf{x}, \pi) \setminus \partial x_1}{\partial V(\mathbf{x}, \pi) \setminus \partial x_s} = \frac{\pi_s \cdot p_1}{\pi_1 \cdot p_s}. \quad (3)$$

Now, we have

$$\frac{\partial V(\mathbf{x}, \pi)}{\partial x_s} = \pi_s \left[u_1(x_s, V(\mathbf{x}, \pi)) + u_2(x_s, V(\mathbf{x}, \pi)) \frac{\partial V(\mathbf{x}, \pi)}{\partial x_s} \right] + \sum_{s' \neq s} \pi_{s'} u_2(x_{s'}, V(\mathbf{x}, \pi))$$

which is equivalent to

$$\frac{\partial V(\mathbf{x}, \pi)}{\partial x_s} = \pi_s u_1(x_s, V(\mathbf{x}, \pi)) \cdot \left(1 - \sum_{s=1}^S \pi_s u_2(x_s, V(\mathbf{x}, \pi)) \right)^{-1}$$

Hence, for any $s, s' \in \{1, 2, \dots, S\}$

$$\text{MRS}_{s,s'} = \frac{\pi_s u_1(x_s, V(\mathbf{x}, \pi)) \cdot \left(1 - \sum_{s=1}^S \pi_s u_2(x_s, V(\mathbf{x}, \pi)) \right)^{-1}}{\pi_{s'} u_1(x_{s'}, V(\mathbf{x}, \pi)) \cdot \left(1 - \sum_{s=1}^S \pi_s u_2(x_s, V(\mathbf{x}, \pi)) \right)^{-1}} = \frac{u_1(x_s, V(\mathbf{x}, \pi))}{u_1(x_{s'}, V(\mathbf{x}, \pi))}.$$

For $s = 2$, we can rewrite (3) as

$$\frac{u_1(x_1, V(\mathbf{x}, \pi))}{u_1(x_2, V(\mathbf{x}, \pi))} = \frac{\pi_2 \cdot p_1}{\pi_1 \cdot p_2} \quad (4)$$

Given the uniqueness of $V(\cdot, \cdot)$ in the implicit representation (1) (see Dekel (1986) p. 314), equation (4) allows us to solve for $V(\mathbf{x}, \pi)$. Thus, for any other $s \in \{3, 4, \dots, S\}$, it holds that the expression

$$\frac{u_1(x_1, V(\mathbf{x}, \pi))}{u_1(x_s, V(\mathbf{x}, \pi))} = \frac{\pi_s \cdot p_1}{\pi_1 \cdot p_s}$$

can be solved for x_s , and using the budget constraint, it yields a unique solution to the optimization problem (2) requiring the existence of a function $f : \mathbb{R}_{++}^3 \rightarrow \mathbb{R}$ such that

$$x_s = f(x_1, x_2, k_s) \quad \text{where} \quad k_s := \frac{\pi_1 p_s}{\pi_s p_1} \quad (5)$$

with the desired properties. Note that the dependence of x_s on x_2 comes from (4). \square

Corollary 3. *A functional restriction that is a necessary condition for a preference \succsim to be in the betweenness class is as follows. Let $\vec{p}_{1,2}^{\leftarrow s}$ be a direction of price changes where p_s is constant and x_1, x_2 are unchanging. The directional derivative of x_s in this direction must be 0. Formally:*

$$\mathcal{R}_{bet}(\succsim)(p) = \frac{\partial x_s}{\partial \vec{p}_{1,2}^{\leftarrow s}} \Big|_{x_1, x_2} = 0 \quad \forall s \neq s' \in \{3, 4, \dots, S\},$$

where this derivative is taken in directions where x_1, x_2 are not changing.

Corollary 4. *Let $\mathcal{U} \subset \mathcal{M}$ be the class of preferences characterized by \mathcal{R}_{bet} , i.e.*

$$\succeq \in \mathcal{U} \iff \succeq \in \mathcal{R}_{bet}.$$

The class of betweenness preferences is a subset of \mathcal{U} .

This gives us a one-sided test of betweenness preferences which is weaker than a test of expected utility provided by [Kubler et al. \(2014\)](#). A test that is generated from the above restrictions will never reject demands from the betweenness class; of course, the other direction of the implication is open, so we may not reject all preferences that are not from the betweenness class.

5 Implementation

This section assesses the empirical performance of our tests in the case of homothetic and weakly separable preferences.

5.1 Simulations

In what follows, we approximate the class of rational preferences \mathcal{M} using AIDS as the resulting demand functions correspond to first-order approximations to any demand function derived from utility maximization ([Deaton and Muellbauer, 1980](#)). Let K denote the number of goods and I denote income. The almost ideal demand functions when there are two goods are given by

$$\begin{aligned} \frac{x_{1,t}}{I} &= \alpha_1 + \gamma_{1,1} \log(p_{1,t}) + \gamma_{1,2} \log(p_{2,t}) + \beta_1 \log(I/P_t) \\ \frac{x_{2,t}}{I} &= \alpha_2 + \gamma_{2,1} \log(p_{1,t}) + \gamma_{2,2} \log(p_{2,t}) + \beta_2 \log(I/P_t), \end{aligned}$$

where

$$P_t = \exp \left(\sum_k \alpha_k \log(p_{k,t}) + \frac{1}{2} \sum_j \sum_k \gamma_{k,j} \log(p_{k,t}) \log(p_{j,t}) \right).$$

We impose the following restrictions on demand parameters to ensure demands satisfy adding up, Slutsky symmetry, and are homogeneous of degree zero in prices

and expenditure:

$$\sum_i \alpha_i = 1, \sum_i \gamma_{i,j} = 0, \sum_i \beta_i = 0, \sum_j \gamma_{i,j} = 0, \gamma_{i,j} = \gamma_{j,i}.$$

We check the effectiveness of our approach to correctly reject a data generating process outside a class of interest. We first consider the class of homothetic preferences ($\beta = 0$). The data generating processes are given by almost ideal demand functions with varying strengths of income effects as captured by β .²¹ Results are presented in Table 1.

Table 1: Homotheticity

β	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
ϵ_k	1.0	1.0	1.0	1.0	0.33
δ_k	0.05	0.05	0.05	0.05	0.016
$n(\epsilon_k, \delta_k)$	60	60	60	60	100
T_n	8.84	7.97	7.71	5.80	0.41

Table 1 shows that our method quickly rejects ($TS_n > \epsilon_k$) a dataset whose demands are not homothetic with a low probability (δ_k) of falsely rejecting the null hypothesis. The number of observations (n) needed to correctly reject the null hypothesis only needs to be larger when the data generating process is extremely close ($\beta = 10^{-5}$) to the class of homothetic preferences.

Next, we consider the null hypothesis that observed demands are generated by weakly separable preferences. The data generating processes are given by an almost ideal demand function with $\beta = 0$ and varying $\gamma_{i,j}$, where $i \in g_1$ and $j \in g_2$. Note that the data generating process would be consistent with weak separability if $\gamma_{1,2} = \gamma_{1,3} = 0$. Results are presented in Table 2.

Table 2: Weak Separability

$\gamma_{1,j}$	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}
ϵ_k	1.0	1.0	1.0	1.0	1.0
δ_k	0.05	0.05	0.05	0.05	0.05
$n(\epsilon_k, \delta_k)$	60	60	60	60	60
T_n	19.31	19.31	19.31	19.30	17.54

²¹Note that β is the only parameter that affects the functional restriction for the class of homothetic preferences.

Table 2 shows that the test correctly rejects demand observations that are not generated by weakly separable preferences without any difficulty. Additional details about the simulations are provided in the Appendix.

6 Extensions and Other Properties

6.1 Measurement Error

The previous results have shown how to test restrictions on demands when we perfectly observe choices made by an agent. This section shows that additive measurement error does not affect the validity of our procedure under reasonable assumptions. We appeal largely to Bartlett et al. (1994), but adapt their assumptions to our specific setting.

Let \mathcal{D} be the class of distributions with mean 0 and variance bounded above by some uniform bound $V_{\mathcal{D}}$. Further, assume that every distribution $D \in \mathcal{D}$ has a uniformly continuous cumulative density function.

Definition 7. A distribution of measurement error is *admissible* if it belongs to \mathcal{D} .

Theorem 6.1. (*Bartlett et al., 1994, Theorem 24*) Suppose that we have a class of functions $\mathcal{F} : \mathcal{X} \rightarrow [0, 1]$ that is learnable. Also, suppose the data are subject to admissible measurement error such that the analyst observes $\{f(x_i) + \eta_i\}_{i=1}^n$, where η_i is an i.i.d. realization from some distribution $D \in \mathcal{D}$. The class \mathcal{F} is learnable.

Corollary 5. *If there is admissible measurement error on individual choices, then the family of income Lipschitz demands is learnable.*

6.2 Choice Set Independence

We finally discuss one more attractive property of our approach. That is, the independence of our procedure from the nature of choice sets observed by the analyst. Note that this is in stark contrast from “revealed preference” tests that do depend on the nature of budget sets and the sufficiency of first-order conditions.²² Unlike the classical literature on revealed preference, we do not use the Lagrangian approach, thus avoiding its limitations. All we require is that the setup we are considering allows for uniform PAC learnability of preferences (or demand functions).

²²See Nishimura, Ok, and Quah (2017); Polisson, Quah, and Renou (2020); Forges and Minelli (2009)

Lemma. *Suppose \mathcal{O} is a family of choice sets and O is an element of \mathcal{O} . Assume a decision maker with preference \succeq generates data of the form $\{O, x(O)\}_{O=1}^n$. If individual demand functions are PAC learnable under the family of budget sets \mathcal{O} . Any restriction \mathcal{R} which can be used as a test with prices can be used to construct a test with the new datasets $\mathcal{C} \subset \mathcal{M}$.*

Proof. The argument is simple: in our procedure, we never used the structure of the data we observed other than using it to generate a candidate demand. By the definition of PAC learnability from [Valiant \(1984\)](#), it must be that for any $\epsilon, \delta > 0$ there is an n such that the probability that generalization error is greater than ϵ is less than δ . This is the only condition we need for our procedure to work. \square

7 Conclusion

This paper introduces an innovative approach to test decision-makers' adherence to specific classes of choice behavior using finite choice data. Traditionally, the revealed preference (RP) approach has been used to conduct such tests. However, the RP approach can be computationally intractable for certain classes of preferences. To address this issue, we propose an alternative method based on preference learnability to adapt tests of demand functions to finite choice data, thus allowing us to exploit the increased simplicity of functional tests.

Our approach shifts the focus from testing the dataset itself to testing the decision maker (DM). Unlike RP tests that always reject a dataset if it cannot be rationalized by any preference from the class of interest, our tests are one-sided: they never reject a rationalizable dataset but may fail to reject a non-rationalizable one. However, the probability of failing to reject a non-rationalizable dataset decreases as more data are sampled. Our simulation exercises show that our tests perform well in finite samples and correctly reject demands that do not belong to specific classes of preferences.

Our procedure is grounded in the Probably Approximately Correct (PAC) learning framework and enables us to construct tests that unite functional and finite data testing approaches. This new method holds promise for a wide range of applications, from weak separability to choice under risk, demonstrating its versatility for the analysis of decision making.

Appendix

This appendix provides additional details about the simulators.

Pseudo-Code

The following pseudo algorithm details the procedure for testing whether $x_{\succeq}(p) \in \mathcal{C}$ in our simulations.

1. Define two sequences $(\epsilon_k)_{k=1}^{\infty} \rightarrow 0$, $(\delta_k)_{k=1}^{\infty} \rightarrow 0$ and let $T_n = 0$.
2. Compute the size of the dataset $n(\epsilon_k, \delta_k)$ required for the observed demand x_{\succeq} and the true demand x_{\succeq}^* to be within ϵ_k from one another with probability $1 - \delta_k$.
3. Draw prices p of dimension $K \times n(\epsilon_k, \delta_k)$ uniformly over an arbitrary interval.
4. Compute the functional restriction at the observed demands $\mathcal{R}(x_{\succeq}(p))$.
5. Compute $\gamma(\cdot)$.
6. Compute $T_n = \gamma\left(\left\{\int_{\mathcal{P}} |\mathcal{R}(\succeq)(p)|^2 dp\right\}^{\frac{1}{2}}\right)$. If $T_n > \epsilon_k$, stop. Else, let $k = k + 1$ and go back to Step 2.

Note that the algorithm can be easily adapted for real datasets. Indeed, Step 3 would be skipped since the distribution of prices is given by the empirical distribution. Also, since the number of observations n is fixed, the algorithm would stop as soon as $T_n > \epsilon_k$ or $n(\epsilon_k, \delta_k) > n$. In the latter case, we would not reject $x_{\succeq}(p) \in \mathcal{C}$.

Approximating \mathcal{C}

Recall that the set of rational demands is approximated by AIDS. In what follows, we detail how we approximate the set of demands that belong to the class of interest \mathcal{C} .

For homothetic preferences, the parameters α_i are drawn uniformly from $[0, 1]$ and the parameters $\gamma_{i,j}$ are drawn uniformly over the support that ensures demands are positive. The parameters β_i determine whether demands are consistent with the functional restriction for homotheticity, where $\beta_i = 0$ if and only if $\mathcal{R}(\succeq) = 0$. The latter motivates approximating the class of homothetic preferences \mathcal{C} by AIDS demands with the additional restriction $\beta = 0$.

For weakly separable preferences, [Moschini, Moro, and Green \(1994\)](#) provide restrictions on AIDS demands to satisfy weak separability. In the case of two groups, weak separability requires

$$\frac{\gamma_{i,k} + x_i x_k + \beta_i \beta_k + \log(I/P_t)}{\gamma_{j,m} + x_j x_m + \beta_j \beta_m + \log(I/P_t)} = \frac{(x_i + \beta_i)(x_k + \beta_k)}{(x_j + \beta_j)(x_m + \beta_m)},$$

where $(i, j) \in g_1$ and $(k, m) \in g_2$. This condition holds globally if $\beta_i = \beta_j = \beta_k = \beta_m = 0$ and $\gamma_{i,l} = \gamma_{j,m} = 0$. In our simulations, we consider data generating processes that deviate from those restrictions.

The functional restriction for weak separability is not well-defined when preferences are homothetic in the class of almost ideal demands. Indeed, in this special case we have $S_{ij} = 0$ for all $i \in g_1$ and $j \in g_2$. Thus, we define the functional restriction for homothetic weak separability by the average absolute deviation:

$$\mathcal{R}(\succ)(p) = \frac{1}{|g_1 + g_2|} \sum_{i \in g_1} \sum_{j \in g_2} \text{abs}(S_{i,j}),$$

where $|\cdot|$ denotes the cardinality of the separable group.

Dataset Size

Let L denote the income-Lipschitz constant for the class of demands \mathcal{M} . Let $\text{fat}_{\mathcal{M}}$ denote the fat-shattering dimension of \mathcal{M} . [Beigman and Vohra \(2006\)](#) show that $\text{fat}_{\mathcal{M}}(\epsilon) \leq \left(\frac{L}{\epsilon}\right)^K$ and that the true demand function can be learned (up to ϵ - δ accuracy) with a number of observations $n(\epsilon, \delta) = O\left(\frac{1}{\epsilon^2} \left(\log^2\left(\frac{1}{\epsilon}\right) \text{fat}_{\mathcal{M}}(\epsilon) + \log\left(\frac{1}{\delta}\right)\right)\right)$, where $\epsilon, \delta > 0$. With the class of rational preferences approximated by AIDS, $\frac{\partial x(p)}{\partial I} = \beta/I$ such that $L = \text{abs}(\beta/I)$. This gives the fat-shattering dimension and allows us to compute $n(\epsilon, \delta)$ up to some scale $C > 0$. In our simulations, we set the scaling of $n(\epsilon, \delta)$ to 20.

Gamma function

For S pairs $(\succ, \succ^*) \in \mathcal{C} \times \mathcal{M}$, compute $\text{erf}(\succ, \succ^*) = \left(\sum_{k,t} \frac{1}{KT} (x_{k,t}(\succ, p) - x_{k,t}(\succ^*, p))^2\right)^{0.5}$ and the functional restriction $R(\succ)(p) := \int_{\mathcal{P}} |\mathcal{R}(x_{\succ}(p))|^2 dp = \left(\sum_{k,t} \frac{1}{KT} \mathcal{R}(x_{k,t}(\succ, p))^2\right)^{0.5}$. Let \mathcal{A} denote the set of all pairs (\succ, \succ^*) such that $\mathcal{R}(\succ)(p) \leq \epsilon_k$. Define $\gamma(\epsilon_k) = \max_{(\succ, \succ^*) \in \mathcal{A}} \text{erf}(\succ, \succ^*)$. In the case of AIDS, we have $\mathcal{R}(x_{k,t}(\succ, p)) = -\beta_k$.

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